6.2. Annex 2: Method 2 - Nutrient mass balance models for Lakes, a Danish approach

Mass balance models for lakes were developed by Vollenweider & Keerekes (1982) for phosphorus and Kelly *et al.* (1982) for nitrogen (see also Equation (4)). These Models can be applied, in particular for phosphorus, in cases where the residence time of the water in the lake or lakes is known. In Denmark special mass balance models have been developed for the specific conditions of shallow lakes.

2A: The Danish model for Phosphorus retention in shallow lakes

The model "P2" has only two state variables: total phosphorus in lake water (P_l) and exchange-able total phosphorus in sediment (P_s). The driving variables in the model are the monthly inlet concentration of total phosphorus (P_i), the corresponding monthly water discharge (Q) and the lake water temperature (T).

In Danish streams the fraction of the total phosphorus transport contributed by the particulate total phosphorus transport are generally high (Svendsen *et al*., 1995; Kronvang & Bruhn, 1996). The proportion of the particulate total phosphorus transport in such streams increases with increasing water discharge in the individual stream (Svendsen *et al*., 1995; Kronvang & Bruhn, 1996). The input of particulate phosphorus will settle instantly in the lakes and therefore not contribute to the phosphorus pool in the lake water immediately. In the P2-model, this is simulated by dividing the input of total phosphorus between the lake water pool of phosphorus and the sediment pool of total phosphorus. The fraction of the total phosphorus input forwarded to the lake water pool is given by the factor 'k', thus the fraction of the total phosphorus input forwarded to the sediment pool is given by '1-k'. The actual values of 'k' have been related to water discharges by the following empirical relation:

$$k = \frac{1}{1 + \sqrt{\frac{V}{365 \cdot Q}}}$$

k = Fraction of input total phosphorus input to lake water,
(1-k) = Fraction of input total phosphorus input to sediment.
Q = Water discharge (m^3/day) and
V = Lakewater volume (m^3).

In P2 the use of k is optional. If required, all the total phosphorus input is considered to be input to the lake water pool of total phosphorus. This might prove useful when considering lakes where most of the input is dissolved phosphorus.

The dynamics of lake water and total phosphorus is given by the difference between input and output. The sedimentation of total phosphorus is deducted, and the release of total phosphorus from the sediment is added, see below:

$$\frac{dP_l}{dt} = \frac{Q}{V} \cdot (k \cdot P_i - P_l) - SED + REL$$

P_l = Lake water total phosphorus concentration (g/m^3),
P_i = Inlet total phosphorus concentration (g/m^3),
SED = Gross Sedimentation of total phosphorus from lake water to sediment (g P/ ($m^2 \cdot$ day),
REL = Release of total phosphorus from sediment to lake water (g P/ ($m^2 \cdot$ day).

Correspondingly, the change of total phosphorus in the sediment is given by the partial input from the inlet. The sedimentation of total phosphorus is added, and the release of total phosphorus from the sediment is deducted.

$$\frac{dP_s}{dt} = \frac{Q}{V} \cdot ((1-k) \cdot P_i) + SED - REL$$

P_s = Sediment total phosphorus concentration (g/m^3, in output converted to g/m^2).

The sedimentation of total phosphorus is given by a constant sedimentation rate, times the lake water pool of total phosphorus. In order to have the same units of sedimentation in the different lakes, the equation is adjusted by the lake mean depth. The temperature dependence of this process is modelled by a standard Van Hoff's equation as follows:

$$SED = bS \cdot (1+tS)^{T-20} \cdot \frac{P_l}{Z}$$

bS = Sedimentation rate of total phosphorus,
tS = Temperature correction for bS,
T = Lake water temperature (°C) and
Z = Mean water depth (m).

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$$REL = bF \cdot (1+tF)^{T-20} \cdot P_s$$

bF = Release rate of total phosphorus and
tF = Temperature correction for bF.

An estimate of lake retention of total phosphorus is thus given by the difference of sedimentation and release (SED-REL). The calibration of the parameters were done on an eight years series of monthly data on water balance and phosphorus mass balances from 16 Danish lakes; some of these lakes were permanently stratified during summer. They were all quite shallow with a mean depth below 10m and a max depth below 22 m (see Table 2.1).

Table 6.2: Characteristics of the 16 lakes.

	Lake area (km²)	Mean depth (m)	Max depth (m)	Water retention time (days)	Inlet P-konc. (year) (mg P/l)	Lake P-conc. (year) (mg P/l)	Lake P-conc. (summer) (mg P/l)	Chlorophyll a (summer) (μg/l)	Secchi depth (summer) (m)
Min	0,05	0,9	1,5	7	0,090	0,094	0,086	38	0,4
Median	0,34	1,8	3,2	30	0,220	0,148	0,286	113	0,6
Mean	0,91	2,5	5,3	70	0,249	0,211	0,322	132	0,8
Max	6,62	9,9	21,7	266	0,849	0,963	0,991	350	2,0

The results of the calibration of the model on the basis of the data from the 16 lakes is given in Table 2.2.

Table 6.3: Calibrated parameter values for the P2-model.

Parameter	Calibrated value
Sedimentation rate (bS)	0,0470
Temperature dependence of P-sedimentation (tS)	0,0000
Sediment release rate (bF)	0,000595
Temperature dependence of P-release (tF)	0,0800

As a consequence of the value of tS calibrates to 0, the equation for the sedimentation can be reduced to:

$$SED = bS \cdot \frac{P_l}{Z}$$

The initial values of PS (t=0) were calibrated for each lake, reflecting differences in the development of phosphorus loading in the past years and, consequently, in the actual sediment pool of total phosphorus. Since the model is calibrated by using shallow Danish lakes, caution should be applied when using the model under other circumstances, especially if the lake characteristics differ (see Table 2.1). The most crucial factor is stratification, as the model will not perform very well for permanently stratified lakes.

2B: The Danish nitrogen model for lakes (Jensen *et al.* 1994)

The aim of this study was to elucidate the seasonal dynamics of nitrogen retention in lakes differing in hydraulic and N loading. In addition, besides the annual models, the first simple model capable of accurately predicting seasonal variation in lake water concentration of total nitrogen and retention of total nitrogen is presented.

The model of lake retention of total nitrogen on a monthly basis is given by:

$$N_{ret}(\%) = a*q^{(T-20)} * N_{retmax}$$

T = water temperature; and

N_{retmax} = given by the sum of the inflow of total nitrogen and the pool of total nitrogen in the lake water.

The parameters have been calibrated to 0.455 and θ to 1.087 on the basis of data from 16 shallow Danish lakes (Windolf *et al.* 1996).

The model of lake retention of total nitrogen on a yearly basis is given by:

$$N_{ret}(\%) = a*tw^{b}$$

The parameters have been calibrated to 78 and b to 0.48 on the basis of data from 16 shallow Danish lakes (Windolf *et al.* 1996). The calibration of the parameters was done on a 3-4 years series of monthly data on water balance and phosphorus mass balances from 16 Danish lakes. Some of these lakes were permanently stratified during summer and they were all quite shallow (mean depth below 6m), see Table 2.3.

Table 6.4: Characteristics of the 16 lakes.

Lake	z (m)	TP (ug P/l)	Chla (ug/l)	Secchi depth (m)	TN (mg N/l)	N02 + N03 (mg N/l)	n
Vesterborg sø	1.4	241 (27)	105	0.70 (0.04)	5.21 (0.45)	3.70 (0.55)	4
Søgård sø	1.6	272 (34)	153	0.58 (0.05)	6.69 (0.61)	4.67 (0.79)	4
Lemvig sø	2.0	239 (11)	45 (4)	0.74 (0.05)	4.30 (0.48)	3.10 (0.36)	4
Hejrede sø	0.9	123 (6)	75 (10)	0.65 (0.05)	4.34 (0.29)	2.18 (0.36)	4
FuglesØ	2.0	256 (22)	75 (4)	1.12 (0.03)	4.18 (0.37)	2.39 (0.41)	3
Fårup sø	5.6	92 (5)	37 (4)	1.77 (0.08)	1.51 (0.05)	0.79 (0.02)	4
Langesø	3.1	279 (30)	62 (8)	1.24 (0.04)	3.80 (0.15)	2.39 (0.13)	4
Kilen	2.9	187 (17)	103 (22)	0.68 (0.09)	2.17 (0.07)	0.76 (0.06)	4
Jels Oversø	1.2	273 (26)	100 (12)	0.85 (0.05)	6.90 (0.18)	5.34 (0.26)	3
Ørn Sø	4.0	108 (2)	36 (2)	1.57 (0.05)	1.43 (0.04)	0.55 (0.02)	4
Hinge Sø	1.2	122 (3)	90 (9)	0.68 (0.03)	4.44 (0.20)	2.95 (0.17)	4
Dons Nørresø	1.0	216 (29)	251	0.56 (0.04)	5.05 (0.08)	3.05 (0.11)	4
Borup Sø	0.9	150 (10)	78 (9)	0.92 (0.04)	4.93 (0.46)	2.97 (0.36)	4
Gundsømagle	1.2	1127	276	0.55 (0.02)	5.92 (0.42)	2.85 (0.44)	4
Store Søgård	2.7	465 (53)	41 (1)	0.79 (0.05)	6.27 (0.32)	3.33 (1.65)	3
Bryrup	4.6	107 (7)	33 (4)	2.10 (0.10)	4.15 (0.11)	3.10 (0.11)	4

Since the model is calibrated on shallow Danish lakes, caution should be applied when using the model under other circumstances, especially if the lake characteristics differ (see Table 6.2). The most crucial factor is stratification, since the model will not perform very well for permanently stratified lakes.